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Minimal mass matrices for Dirac neutrinos

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Abstract: We consider the possibility of neutrinos being Dirac particles and study minimal mass matrices with as much zero entries as possible. We find that up to 5 zero entries are allowed. Those matrices predict one vanishing mass state, CP conservation and U_{e3} either zero or proportional to R, where R is the ratio of the solar and atmospheric Δm^2. Matrices containing 4 zeros can be classified in categories predicting $U_{e3} = 0$, $U_{e3} \neq 0$ but no CP violation or $|U_{e3}| \neq 0$ and possible CP violation. Some cases allow to set constraints on the neutrino masses. The characteristic value of U_{e3} capable of distinguishing some of the cases with non-trivial phenomenological consequences is about $R/2\ \sin 2\theta_{12}$. Matrices containing 3 and less zero entries imply (with a few exceptions) no correlation for the observables. We outline models leading to the textures based on the Froggatt-Nielsen mechanism or the non-abelian discrete symmetry $D_4 \times Z_2$.

Keywords: Discrete and Finite Symmetries, Solar and Atmospheric Neutrinos, Neutrino Physics, Beyond Standard Model.

Contents

1. Introduction

Recent years witnessed a dramatic increase of our knowledge about the properties of neutrinos (for recent reviews, see for instance [1]). The experimental evidence showed that at least two neutrinos possess a non-vanishing rest mass, whose value is bounded from above by roughly 1 eV [2, 3]. This implied smallness of neutrino masses is commonly attributed to the see-saw mechanism [4], which explains the tiny neutrino masses through their inverse proportionality to the mass scale of heavy Majorana neutrinos. In addition, the light neutrinos are predicted to be of Majorana type. The smoking-gun signature of this property would be the observation of neutrinoless double beta decay [3]. There is currently no incontrovertible evidence for this process (see the discussion in [3]) and therefore the possibility that neutrinos are Dirac particles, just like all the other known fermions, has to be considered as well. If neutrinos are indeed Dirac particles, their mass term is given just by the coupling of the lepton doublets with the Higgs doublet and a right-handed neutrino

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Standard Model singlet state. One should then explain however why the corresponding Yukawa coupling is so much smaller than the Yukawa coupling for the other fermions. There are several possibilities for this to arise. All have in common that effectively there is an additional $\mathrm{U}(1)_{\mathrm{B-L}}$ symmetry which forbids a Majorana mass term. For instance:

- in theories with large extra dimensions [5] the effective Yukawa coupling is determined by the overlap of the wave functions of the SM particles (residing in the usual 3 + 1 dimensions) with the wave functions of the right-handed singlet neutrinos. The latter propagate in additional dimensions (the bulk) and a volume suppression of order M_F/M_{Pl} occurs, where M_{Pl} is the Planck scale and M_F the fundamental gravity scale, possibly as low as 10 TeV. Similar volume suppression can occur in models with warped extra dimensions or "fat brane" models [6]. For more details of these possibilities, see the recent reviews in [7];
- in supersymmetric frameworks the suppression factor might be given by the SUSY breaking scale divided by the Planck scale in scenarios with anomaly mediated supersymmetry breaking [8]. For other supersymmetry or supergravity models of Dirac neutrino masses, see, e.g., [9];
- in superstring models it is possible to obtain small neutrino masses through a hierarchy of vacuum expectation values of SM singlets [10]. In this respect it is interesting to note that in string theories it seems difficult to obtain a simple see-saw mechanism [11];
- if neutrinos are massless at tree level, small Dirac masses might be generated through radiative mechanisms, so that the smallness is explained by loop suppression [12];
- other approaches to generate Dirac neutrino masses are discussed in refs. [13].

A common argument in favor of Majorana neutrinos is that the implied lepton number violation can be used to generate the baryon asymmetry of the universe via the leptogenesis mechanism [14]. However, even with Dirac neutrinos it is possible to achieve this [15, 16]. We therefore conclude that there is some motivation for studying Dirac neutrinos.

Once the Dirac nature of neutrinos is accepted, one can ask immediately what the structure of the neutrino mass matrix m_ν looks like. In particular, minimal mass matrices are of interest since they usually imply interesting and testable correlations between the observables. For Majorana neutrinos, there has been some interest in texture zeros of m_ν [17]. It has been found that only two independent zero entries are allowed by current data [17]. Models leading to the successful textures [18] as well as renormalization effects [19] have been studied. Here we perform a similar analysis for Dirac neutrino mass matrices. Since Dirac mass matrices are — in contrast to Majorana mass matrices — in general not symmetric, the results are expected to differ. We shall find that one is able to put up to 5 zero entries in a Dirac neutrino mass matrix. Those cases predict one vanishing neutrino mass, CP conservation and small or zero U_{e3}. Already for matrices with 4 zeros there are successful cases in which there is no correlation between the observables. Some of them display however a correlation different from the 5 zero cases.

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We also outline models based on the discrete symmetry group $D_4 \times Z_2$ or on the Froggatt-Nielsen mechanism, which allow in principle to reproduce the textures under consideration.

The paper is structured as follows: section 2 contains several general considerations about Dirac mass matrices, their form as implied by current data and comments on the possibility to determine the nature of the neutrino. Then we perform a systematic search for zero textures for Dirac neutrinos in section 3, discussing the cases of 5, 4 and 3 or less zeros. We comment on the implications of the Dirac mass matrices being symmetric. Models based on the flavor symmetry $D_4 \times Z_2$ and on the Froggatt-Nielsen mechanism are outlined in section 4, before we conclude in section 5.

2. Dirac neutrino mass matrices: general considerations

For Dirac neutrinos, the PMNS [20] matrix can be parametrized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\, e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\, e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\, e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\, e^{-i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.1}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and δ denotes the CP phase, sometimes called "Dirac-phase". Global analyzes of current data allow for the following values of the observables [21]:

$$\begin{aligned} &\sin^2\theta_{12} = 0.30^{+0.04}_{-0.05}\,, \qquad \sin^2\theta_{23} = 0.50^{+0.14}_{-0.12}\,, \qquad \sin^2\theta_{13} = |U_{e3}|^2 \leq 0.031 \\ &\Delta m^2_{21} \equiv \Delta m^2_\odot = \left(7.9^{+0.6}_{-0.6}\right)\cdot 10^{-5}\,\mathrm{eV}^2\,, \qquad \left|\Delta m^2_{31}\right| \equiv \Delta m^2_\mathrm{A} = \left(2.2^{+0.7}_{-0.5}\right)\cdot 10^{-3}\,\mathrm{eV}^2 \\ &\Rightarrow \quad R \equiv \frac{\Delta m^2_\odot}{\Delta m^2_\mathrm{A}} = 0.036^{+0.014}_{-0.011}\,, \end{aligned} \tag{2.2}$$

where Δm^2_{ij} denotes $m_i^2 - m_j^2$. We gave the best-fit values and the allowed 2σ ranges. Regarding the mass ordering, both the normal and the inverted ordering can be accommodated, corresponding to the cases $m_3 > m_2 > m_1$ and $m_2 > m_1 > m_3$, respectively. The extreme cases are the normal hierarchy (NH), which is defined as $m_3^2 \simeq \Delta m^2_\mathrm{A} \gg \Delta m^2_\odot \simeq m_2^2 \gg m_1^2$, and the inverted hierarchy (IH), which is defined as $m_2^2 \simeq m_1^2 \simeq \Delta m^2_\mathrm{A} \gg m_3^2$. The quasi-degenerate spectrum (QD) occurs when the common neutrino mass scale m_0 is much larger than the mass splittings, i.e., $m_0^2 \equiv m_3^2 \simeq m_2^2 \simeq m_1^2 \gg \Delta m^2_{\odot,\mathrm{A}}$.

Mass matrices for Dirac neutrinos are not symmetric. Hence, they are parametrized by nine complex variables $a, \ldots, l$:

$$m_\nu = \begin{pmatrix} a & b & d \\ e & f & g \\ h & k & l \end{pmatrix} = U\, m_\nu^\mathrm{diag}\, V^\dagger\,, \tag{2.3}$$

where U, V are unitary matrices and m_ν^diag is a diagonal mass matrix containing the three mass eigenstates. In the basis in which the charged lepton mass matrix is diagonal the relevant lagrangian reads

$$\mathcal{L} = -\overline{(\nu'_\ell)_L}\, m_\nu\, (\nu'_\ell)_R - \overline{\ell_L}\, m_\ell^\mathrm{diag}\, \ell_R + \frac{g}{\sqrt{2}} W_\mu\, \overline{\ell_L}\, \gamma^\mu (\nu'_\ell)_L + \mathrm{h.c.}\,, \tag{2.4}$$

where $\ell = e, \mu, \tau$. After substituting $(\nu'_\ell)_L = U\,(\nu_\ell)_L$ and $(\nu'_\ell)_R = V\,(\nu_\ell)_R$ we identify the PMNS matrix U. This matrix is associated with the diagonalization of $m_\nu\, m_\nu^\dagger$, i.e.,

$$U^\dagger\, m_\nu\, m_\nu^\dagger\, U \equiv U^\dagger\, h\, U = (m_\nu^{\rm diag})^2\,. \tag{2.5}$$

Here we also defined the matrix $h = m_\nu\, m_\nu^\dagger$. The matrix V diagonalizes $m_\nu^\dagger\, m_\nu$ and is, in the absence of right-handed currents, unobservable. In case of Majorana neutrinos, we would have $V = U^*$.

One of the most interesting aspects of neutrino physics is the possibility of leptonic CP violation. Whether a given neutrino mass matrix implies CP violation in oscillation experiments (note that due to the Dirac nature assumed in this work there are no so-called Majorana phases [22]) can for instance be determined from the characteristic matrix h. In terms of h, one can write the Jarlskog invariant J_{CP} as [23]

$$\begin{aligned} J_{CP} &= -\frac{{\rm Im}(h_{12}\, h_{23}\, h_{31})}{\Delta m^2_{21}\, \Delta m^2_{31}\, \Delta m^2_{32}} \\ &= \frac{1}{8}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \sin 2\theta_{13}\, \cos\theta_{13}\, \sin\delta\,. \end{aligned} \tag{2.6}$$

The second line in eq. (2.6) gives the form of J_{CP} in terms of the parametrization eq. (2.1). Hence, if one of the entries h_{12}, h_{13} or h_{23} vanishes, there is no CP violation in oscillation experiments.

It is helpful to investigate the approximate form of h as implied by current data. We will see below that $\lambda \simeq \theta_C \simeq 0.22$ is a useful parameter to describe the order of magnitude of the entries in m_ν. Since it holds to a good precision that $U_{e2} = \sqrt{1/2}\,(1-\lambda)$ and $R = D\,\lambda^2$ (where D is of order one) [24], we can write in case of normal hierarchy $m_3^2 \simeq \Delta m^2_{\rm A}$, $m_2^2 \simeq \Delta m^2_{\rm A}\, D\, \lambda^2$ and in case of inverted hierarchy $m_1^2 \simeq \Delta m^2_{\rm A}$, $m_2^2 \simeq \Delta m^2_{\rm A}\,(1 + D\,\lambda^2)$. Ignoring the smallest mass state and the CP phase and setting for instance $U_{e3} = A\,\lambda^2$ and $U_{\mu 3} = \sqrt{1/2}\,(1 - B\,\lambda^2)$, where A, B are real parameters of order one, we have in case of a normal hierarchy:

$$h \sim \Delta m^2_{\rm A} \begin{pmatrix} \lambda^2 & \lambda^2 & \lambda^2 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix}, \tag{2.7}$$

and for the inverted hierarchy:

$$h \sim \Delta m^2_{\rm A} \begin{pmatrix} 1 & \lambda^2 & \lambda^2 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix}, \tag{2.8}$$

where the entries are to be understood as order-of-magnitude-wise. These two simple matrices are a very helpful guiding line in the search for promising candidates.

Turning to the quasi-degenerate scheme, and in case of a normal ordering, we can write $m_3^2 = m_0^2$, $m_2^2 = m_0^2\,(1 - \eta(1 - D\,\lambda^2))$ and $m_1^2 = m_0^2\,(1-\eta)$, where $\eta \equiv \Delta m^2_{\rm A}/m_0^2$. Then, with the above definition of the mixing parameters, we find

$$h \sim m_0^2 \begin{pmatrix} 1+\eta & \eta\,\lambda^2 & \eta\,\lambda^2 \\ \cdot & 1+\eta & \eta + \lambda^2 \\ \cdot & \cdot & 1+\eta+\lambda^2 \end{pmatrix}. \tag{2.9}$$

A very similar expression is found for the inverted mass ordering. Note that the entries of order "1" in the above matrix are in our approximation exactly 1. This means, that the parameters in m_ν are such that the leading terms in the diagonal entries of h are identical.

We end our general considerations about Dirac mass matrices with the prospects of distinguishing Dirac from Majorana neutrinos. Unfortunately, proving that neutrinos are Dirac particles is much more difficult than proving them to be Majorana particles. The only known process sensitive to the lepton number violation characteristic for light Majorana neutrinos is neutrinoless double beta decay [3]. This process is triggered by the so-called effective mass $\langle m \rangle$, the ee entry of the Majorana mass matrix in the charged lepton mass basis (for details, see, e.g. the recent reviews [3, 25]).

Suppose first that (except for the oscillation results) there is no signal indicating neutrino mass from direct laboratory searches or cosmological observations, i.e., we do not know the exact neutrino mass scale. If the mass ordering is shown to be inverted, we know that — if neutrinos are Majorana particles — there is a lower limit on the effective mass [3, 25] of $\langle m \rangle > \sqrt{\Delta m^2_{\rm A}} \cos^2\theta_{13} \cos 2\theta_{12} \simeq 0.01\,{\rm eV}$. Hence, pushing the limit on the effective mass lower than this value would show that neutrinos are Dirac particles. In case of the normal ordering however, the effective mass can vanish even though neutrinos are Majorana particles. This would indicate that the ee entry of the neutrino (Majorana) mass matrix in the charged lepton mass basis is zero, which would be a very interesting observation by itself. Nevertheless, we have no handle on determining the nature of the neutrino. A possible way out of this dilemma would be to look for analogous processes of neutrinoless double beta decay. However, the associated branching ratios or cross sections are many orders of magnitude below current and future experimental bounds [26].

If we have however a definite value of the neutrino mass scale, be it from cosmology or direct laboratory searches, and do not find a corresponding signal in neutrinoless double beta decay experiments, we could conclude that neutrinos are Dirac particles. To be more quantitative, if the neutrino mass scale is given by m_0, the effective mass should be larger than roughly $m_0 \cos 2\theta_{12}$ if neutrinos are Majorana particles.

3. Minimal Dirac mass matrices

In the remainder of the paper we shall consider minimal Dirac mass matrices, i.e. matrices possessing a certain number of zero entries. In the numerical search for successful matrices, we required the neutrino mixing observables $\sin^2\theta_{12,23,13}$ and $R = \Delta m^2_\odot/\Delta m^2_{\rm A}$ to lie within their 2σ ranges given in eq. (2.2). The non-vanishing entries are varied within two orders of magnitude, i.e., we allow for up to a factor of 100 between them. We checked that the found textures are stable with respect to small perturbations, in the sense that the results stay unchanged as long as the "zero entries" are one order of magnitude smaller than the non-zero entries.

3.1 General considerations

It is fairly easy to see that the possible Dirac nature of neutrinos allows for at least as many zeros as in case of Majorana neutrinos. Consider in eq. (2.3) the case $V = \mathbb{1}$. Then we have $m_\nu = U\, m_\nu^{\rm diag}$ and therefore

$$m_\nu = \begin{pmatrix} U_{e1}\, m_1 & U_{e2}\, m_2 & U_{e3}\, m_3 \\ U_{\mu 1}\, m_1 & U_{\mu 2}\, m_2 & U_{\mu 3}\, m_3 \\ U_{\tau 1}\, m_1 & U_{\tau 2}\, m_2 & U_{\tau 3}\, m_3 \end{pmatrix} . \tag{3.1}$$

In the case of an extreme normal (inverted) hierarchy we can set m_1 $(m_3) = 0$ and thus have three zeros in the first (third) column. Further setting U_{e3} to zero, which is compatible with current data, adds another zero entry to m_ν for the normal hierarchical case. We will indeed encounter successful texture zero matrices with such a structure in the following sections.

One can group Dirac neutrino mass matrices into classes which lead to the same matrix h. Matrices contained in these classes can be transformed into one another by a permutation of the right-handed neutrino fields, which corresponds to an exchange of columns in m_ν, i.e.,

$$m_\nu \to m_\nu' = m_\nu P_i \,. \tag{3.2}$$

This leaves the rest of the lagrangian invariant and leads to the same matrix h because

$$h = m_\nu m_\nu^\dagger \to h' = m_\nu' m_\nu'^\dagger = m_\nu P_i P_i^\dagger m_\nu^\dagger = h \,.$$

The orthogonal permutation matrices P_i are:

$$\begin{aligned} P_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad P_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\ P_4 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad P_5 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad P_6 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{3.3}$$

Note that we cannot put two matrices which are transformed into one another by a transformation of the left-handed lepton fields in one class, since we have already fixed the basis in which the mass matrix of the charged leptons is diagonal and therefore such a transformation would not leave the rest of the lagrangian invariant.

Finally, before proceeding with the search for successful texture zero mass matrices, it is worth commenting on the RG running of a Dirac neutrino mass matrix. The non-diagonal entries in the beta function for the Yukawa couplings are (in the charged lepton mass basis) proportional to [27] $m_\nu\, m_\nu^\dagger/v^2$, where v is the weak scale. In principle, this term could destroy the texture zeros under consideration. However, the smallness of the Yukawa couplings, which are in the SM at most of order 10^{-11} and in the MSSM of order $10^{-11}/\sin\beta$, renders the corrections negligible. Thus, ignoring possible SUSY threshold corrections, the stability of the texture zeros, which will be discussed in the next sections, is to a good precision guaranteed.

3.2 Five texture zeros

When the current neutrino data is taken into account, the maximal number of zeros in a Dirac mass matrix turns out to be 5. There are in total 126 possibilities to put 5 zeros in m_ν, 18 of which allow for a successful explanation of the neutrino data. Their common feature is that their determinant vanishes and therefore they possess one vanishing mass eigenvalue ($m_1 = 0$ for NH and $m_3 = 0$ for IH). Furthermore, there is no CP violation.

These 18 matrices fall into three classes which we denote A, B and $\tilde{B}$. Matrices belonging to class A predict the inverted hierarchy and $U_{e3} = 0$ whereas those belonging to B and $\tilde{B}$ can accommodate both hierarchies and lead in general to non-vanishing U_{e3}.

Class A is represented by the matrix:

$$A = \begin{pmatrix} a & b & 0 \\ 0 & d & 0 \\ 0 & e & 0 \end{pmatrix}, \tag{3.4}$$

which can be transformed into five other matrices with one of the permutation matrices P_i of eq. (3.3):

$$A\,P_2 = \begin{pmatrix} b & a & 0 \\ d & 0 & 0 \\ e & 0 & 0 \end{pmatrix}, \qquad A\,P_3 = \begin{pmatrix} 0 & b & a \\ 0 & d & 0 \\ 0 & e & 0 \end{pmatrix}, \qquad A\,P_4 = \begin{pmatrix} a & 0 & b \\ 0 & 0 & d \\ 0 & 0 & e \end{pmatrix},$$
$$A\,P_5 = \begin{pmatrix} b & 0 & a \\ d & 0 & 0 \\ e & 0 & 0 \end{pmatrix}, \qquad A\,P_6 = \begin{pmatrix} 0 & a & b \\ 0 & 0 & d \\ 0 & 0 & e \end{pmatrix}.$$

Next, we show that class A works only in case of an inverted hierarchy with the additional constraint of $U_{e3} = 0$. Note that the matrices of class A predict for the resulting h a vanishing (23)-subdeterminant, independent of the choice of the variables a, b, d, e. Using the definition $h = U\,(m_\nu^{\rm diag})^2\,U^\dagger$ and requiring that the (23)-subdeterminant vanishes, leads to the condition $\left(m_3^2\,c_{13}^2\left(m_2^2\,c_{12}^2 + m_1^2\,s_{12}^2\right) + m_1^2\,m_2^2\,s_{13}^2\right) = 0$. For a normal hierarchy (i.e., $m_1 = 0$) this gives $m_2^2 m_3^2 c_{12}^2 c_{13}^2 = 0$, which certainly cannot accommodate the data, since it implies $U_{e1} = 0$. In the case of an inverted hierarchy (i.e., $m_3 = 0$) we end up with the equation $m_1^2 m_2^2 s_{13}^2 = 0$, which implies $\theta_{13} = 0$ and leaves the other mixing angles unconstrained.

In general, the formulae for the neutrino mixing observables in terms of $a, \ldots, l$ are quite complicated, but in case of 5 zero entries one can obtain simple expressions. Taking a, b, d, e real and positive we find

$$\tan^2\theta_{23} = \frac{e^2}{d^2}, \qquad \sin^2\theta_{12} = \frac{1}{2}\left(1 + \frac{a^2 + b^2 - d^2 - e^2}{w}\right),$$
$$R = \frac{\Delta m_\odot^2}{\Delta m_{\rm A}^2} = \frac{w}{\frac{1}{2}(a^2 + b^2 + d^2 + e^2 - w)},$$

where

$$w = \sqrt{(a^2 + b^2 + d^2 + e^2)^2 - 4a^2(d^2 + e^2)}\,. \tag{3.5}$$

Typically, a, e and d have (in units of $\sqrt{\Delta m^2_{\rm A}}$) absolute values of order one and b is of order λ^2. With these estimates, one obtains the approximate form of h from eq. (2.8). Note that $\sin^2\theta_{12}$ can give the value $\sin^2\theta_{12} \simeq 0.3$ quite naturally. However, cancellations of order one parameters lead to the smallness of R which induces a certain fine-tuning. This will be the case for most of the solutions which predict the inverted hierarchy. There is typically a large (up to two orders of magnitude) gap between the entries of m_ν and this implies that the inverted hierarchy will — in a bottom-up approach — always be associated with some tuning.

The 12 other matrices able to reproduce the neutrino data can be divided into two very similar classes, which we denote B and $\tilde{B}$. They generate — independent of a, b, d, e — a vanishing h_{12} and h_{13}, respectively. Their representants are

$$B = \begin{pmatrix} 0 & a & 0 \\ b & 0 & 0 \\ d & e & 0 \end{pmatrix} \qquad \text{and} \qquad \tilde{B} = \begin{pmatrix} 0 & a & 0 \\ b & d & 0 \\ e & 0 & 0 \end{pmatrix} . \tag{3.6}$$

Due to the vanishing of h_{12} (h_{13}) in class B ($\tilde{B}$) J_{CP} is also zero, i.e. there is no CP violation. From these conditions one can obtain an interesting correlation between the neutrino mixing observables. Setting the 12 element of $h = U\,(m_\nu^{\rm diag})^2\,U^\dagger$ to zero gives[1]

$$|U_{e3}| = \frac{1}{2}\frac{R\,\sin 2\theta_{12}\,\cot\theta_{23}}{1 \mp R\,\sin^2\theta_{12}} \simeq \frac{1}{2} R\,\sin 2\theta_{12}\,\cot\theta_{23}\,, \tag{3.7}$$

where the $-$ sign is for the normal and the $+$ for the inverted mass ordering. The smallness of R implies that the condition $h_{12} = 0$ gives to a good precision the same correlation for both mass orderings. A similar relation can be deduced from $h_{13} = 0$ for class $\tilde{B}$:

$$|U_{e3}| = \frac{1}{2}\frac{R\,\sin 2\theta_{12}\,\tan\theta_{23}}{1 \mp R\,\sin^2\theta_{12}} \simeq \frac{1}{2} R\,\sin 2\theta_{12}\,\tan\theta_{23}\,. \tag{3.8}$$

The (close-to-)maximal value of θ_{23} indicates that both, $h_{12} = 0$ and $h_{13} = 0$, give approximately the same result for U_{e3}. Moreover, the above relations eqs. (3.7), (3.8) are valid also for a non-vanishing smallest neutrino mass. Consequently, we can apply them also in later sections of this work, when h_{12} or h_{13} vanish for matrices with a non-vanishing determinant. We remark here that one can show that no other element of h except for h_{12} and h_{13} can vanish without leading to conflicts with the neutrino data. We will comment on unsuccessful matrices in section 3.6.

Inserting the best-fit values $R = 0.036$, $\sin^2\theta_{12} = 0.3$ and $\sin^2\theta_{23} = 0.5$, gives for both $h_{12} = 0$ and $h_{13} = 0$ as well as for both mass orderings that $|U_{e3}| \simeq 0.017$ which is about two orders of magnitude below the current 2σ upper bound for $|U_{e3}|^2$. Its value increases (decreases) with increasing (decreasing) $\sin^2\theta_{23}$ for a zero 13 entry in h (and vice versa for zero h_{12}). We plot in figure 1 the correlation between $\sin^2\theta_{23}$ and $|U_{e3}|^2$ for class $\tilde{B}$ in case of the normal hierarchy. Figure 2 displays the same for matrix B, showing the different behavior for these two observables as discussed above and implied by eqs. (3.7), (3.8). The

[1]We have assumed $c_{13} \neq 0$.

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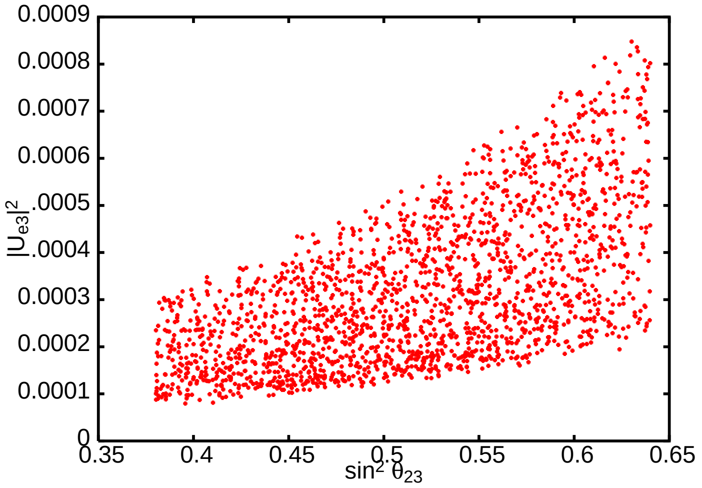

Figure 1: Scatter plot of the correlation between $\sin^2\theta_{23}$ and $|U_{e3}|^2$ for class $\tilde{B}$ in case of normal hierarchy. All matrices associated with a zero entry in h_{13} generate the same behavior.

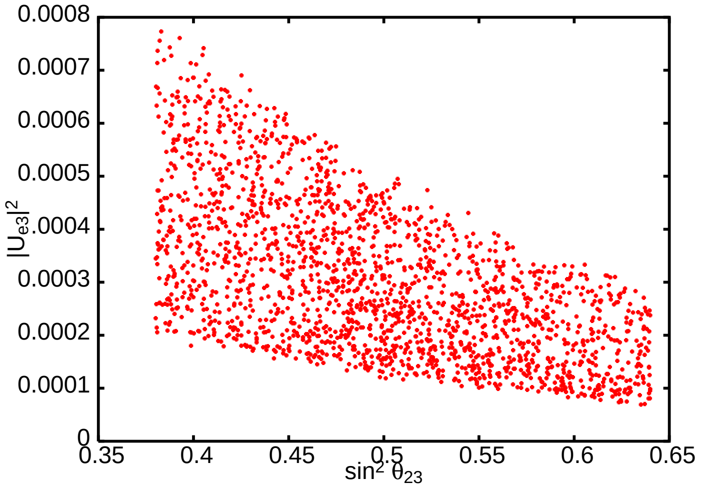

Figure 2: Same as above for class B. All matrices associated with a zero entry in h_{12} generate the same behavior.

respective plots for the inverted hierarchy look identical, as expected from the approximate formulae discussed in this section. As can be seen, $|U_{e3}|^2$ stays between 10^{-4} and 10^{-3}; in particular, for the best-fit value of $\sin^2\theta_{23} = 0.5$ we have $10^{-4} \lesssim |U_{e3}^2| \lesssim 5 \cdot 10^{-3}$.

Now we investigate the dependence of the observables on the variables a, b, d, e with $a, b, d, e > 0$ in class $\tilde{B}$. Similar calculations for the class B reveal similar results and are therefore omitted. In case of normal hierarchy, one finds:

$$\tan^2\theta_{23} = \frac{(a^2 + b^2 + d^2 - e^2 + w)^2}{4\,b^2\,e^2}\,, \qquad U_{e1}^2 = \frac{d^2\,e^2}{d^2\,e^2 + a^2(b^2 + e^2)}\,,$$
$$U_{e3} = \frac{a(a^2 - b^2 + d^2 - e^2 + w)}{\left(4\,b^2\,d^2\,e^2 + a^2(a^2 - b^2 + d^2 - e^2 + w)^2 + d^2(a^2 + b^2 + d^2 - e^2 + w)^2\right)^{1/2}}\,,$$

$$R = \frac{\frac{1}{2}(a^2+b^2+d^2+e^2-w)}{\frac{1}{2}(a^2+b^2+d^2+e^2+w)}\,, \tag{3.9}$$

where

$$w = \sqrt{(a^2+b^2+d^2+e^2)^2 - 4(d^2e^2 + a^2(b^2+e^2))}\,. \tag{3.10}$$

Typically, the normal hierarchy is achieved for parameter values (again in units of $\sqrt{\Delta m^2_{\rm A}}$) of order one for b, e and of order λ for a and d. Inserting those values in m_ν yields the approximate form of h from eq. (2.7), where however the 13 entry vanishes. With the order of magnitude of a, b, d, e we can obtain from eq. (3.9) the following approximate expressions:

$$\begin{aligned}
\tan^2\theta_{23} &\simeq \frac{b^2}{e^2}\,, & \sin^2\theta_{12} &\simeq \frac{\tilde a^2(b^2+e^2)}{\tilde a^2 b^2 + e^2(\tilde a^2 + \tilde d^2)}\,, \\
U_{e3} &\simeq \frac{\tilde a \tilde d\, b}{(b^2+e^2)^{3/2}}\,\lambda^2\,, & R &\simeq \frac{\tilde d^2 e^2 + \tilde a^2(b^2+e^2)}{(b^2+e^2)^2}\,\lambda^2\,,
\end{aligned} \tag{3.11}$$

where we have defined new order one parameters $\tilde a \equiv a/\lambda$ and $\tilde d \equiv d/\lambda$. As can be seen, U_{e3} and R are small because they are proportional to λ^2 and, again, $\sin^2\theta_{12}$ can be naturally of order one.

In case of inverted hierarchy, we get:

$$\begin{aligned}
\tan^2\theta_{23} &= \frac{e^2}{b^2}\,, \qquad U_{e3} = \frac{de}{\sqrt{a^2b^2 + e^2(a^2+d^2)}}\,, \\
\sin^2\theta_{12} &= \frac{1}{2}\left(1 - \frac{e^4 - e^2(a^2 - 2b^2 + d^2) - b^2(a^2 - b^2 - d^2)}{(b^2+e^2)w}\right), \\
R &= \frac{w}{\frac{1}{2}(a^2+b^2+d^2+e^2-w)}\,,
\end{aligned} \tag{3.12}$$

where

$$w = \sqrt{(a^2+b^2+d^2+e^2)^2 - 4(d^2e^2 + a^2(b^2+e^2))}\,. \tag{3.13}$$

In order to explain the neutrino data one needs now that a, b, e are of order one and d of order λ^2 (leading approximately to eq. (2.8) with a vanishing 13 entry). Analyzing eq. (3.12) shows that R owes its smallness to a cancellation of the order one coefficients which is similar to the matrices in class A discussed above.

To conclude this section, both the normal and the inverted hierarchy can be accommodated with Dirac neutrino mass matrices containing 5 zero entries. No observable CP violation is predicted and U_{e3} is either exactly zero (then the inverted hierarchy is present) or very small, corresponding to $|U_{e3}|^2 \simeq 10^{-4} \ldots 10^{-3}$. A non-vanishing value of $|U_{e3}|^2$ above 10^{-3} will therefore rule out any Dirac mass matrix with 5 zero entries, while a value between 10^{-4} and 10^{-3} excludes class A. Class A is also ruled out when the neutrino mass spectrum is found to be normally ordered. The three classes of matrices A, B and $\tilde B$ are furthermore excluded when either leptonic CP violation is present or when a positive signal of neutrino mass in laboratory searches or by cosmological observations is found. Distinguishing B from $\tilde B$ will be very difficult, unless θ_{23} deviates sizably from $\pi/4$.

Class	N	Condition	Phenomenology	Order of magnitude NH	Order of magnitude IH
$A = \begin{pmatrix} a & b & 0 \\ 0 & d & 0 \\ 0 & e & 0 \end{pmatrix}$	6	$\begin{vmatrix} h_{22} & h_{23} \\ h_{32} & h_{33} \end{vmatrix} = 0$	only IH $U_{e3} = 0,\ J_{CP} = 0$	—	$\begin{pmatrix} 1 & \lambda^2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$B = \begin{pmatrix} 0 & a & 0 \\ b & 0 & 0 \\ d & e & 0 \end{pmatrix}$	6	$h_{12} = 0$	$J_{CP} = 0$ $U_{e3} \simeq \frac{R}{2} \sin 2\theta_{12} \cot \theta_{23}$ eq. (3.7), figure 2	$\begin{pmatrix} 0 & \lambda & 0 \\ 1 & 0 & 0 \\ 1 & \lambda & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & \lambda^2 & 0 \end{pmatrix}$
$\tilde{B} = \begin{pmatrix} 0 & a & 0 \\ b & d & 0 \\ e & 0 & 0 \end{pmatrix}$	6	$h_{13} = 0$	$J_{CP} = 0$ $U_{e3} \simeq \frac{R}{2} \sin 2\theta_{12} \tan \theta_{23}$ eq. (3.8), figure 1	$\begin{pmatrix} 0 & \lambda & 0 \\ 1 & \lambda & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & \lambda^2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Table 1: Successful matrices with five zero entries. Shown are a typical representants of the class, the number N of distinct matrices in that class, the characteristic condition giving its phenomenology and an order of magnitude estimate of its entries. We also indicate from which equation the phenomenology results. NH denotes normal hierarchy and IH inverted hierarchy. Note that the smallest mass is always zero because the determinant of m_ν vanishes.

We give in table 1 the classes of matrices which are successful in explaining the neutrino data. Together with the number of members in the class we give their phenomenological consequences and an order of magnitude estimate of the non-zero entries.

3.3 Four texture zeros

As shown in the previous subsection, Dirac mass matrices with five zero entries can accommodate neutrino data, but predict in general vanishing CP violation. To take this effect into account enforces one to allow for an additional non-zero entry in m_ν. As for the case of 5 zero entries, there are 126 possible matrices, 69 of which are successful in explaining the data. Those 69 belong to 12 classes. Among those are 4 classes with a vanishing determinant. From those 4 there are 3 classes which can accommodate leptonic CP violation. Let us start with the CP conserving matrices.

The first class, denoted C and containing 3 members, is represented by

$$C = \begin{pmatrix} a & b & d \\ e & 0 & 0 \\ f & 0 & 0 \end{pmatrix} . \tag{3.14}$$

The resulting structure of h is analogous to the one of class A in the previous subsection, hence one massless neutrino, zero U_{e3} and the inverted hierarchy are predicted. At least one of the entries has to be suppressed by λ^2 in order to reproduce the approximate texture of eq. (2.8).

Other classes have a vanishing 12 or 13 entry of h and therefore, as argued above, a vanishing Jarlskog invariant. The resulting correlations between the observables, as given

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in eqs. (3.7), (3.8) and shown in figures 1 and 2, hold again in these cases. Consider the following 18 matrices, which fall into the three classes $D_{1,2,3}$ and predict $h_{12} = 0$:

$$D_1 = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ d & e & f \end{pmatrix}, \qquad D_2 = \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ e & 0 & f \end{pmatrix}, \qquad D_3 = \begin{pmatrix} a & b & 0 \\ 0 & 0 & d \\ e & 0 & f \end{pmatrix}. \tag{3.15}$$

From eq. (3.7), one has $|U_{e3}| \simeq \frac{1}{2} R \sin 2\theta_{12} \cot \theta_{23}$, whose behavior is displayed in figure 2. To accommodate a normal hierarchical spectrum, the second class D_2 has typically a, b, e of order λ and d, f of order one. If e is of order λ^2, d of order λ and the other parameters of order 1, it turns out that the spectrum can become quasi-degenerate. This can be seen from the fact that the determinant of the matrix, $\det D_2 D_2^\dagger = a^2 b^2 f^2$ is then close to the third power of the trace, $\mathrm{Tr}\, D_2 D_2^\dagger = (a^2 + b^2 + d^2 + e^2 + f^2)$. Recall that in a bottom-up approach, the case of quasi-degenerate neutrinos is fine-tuned, since the diagonal elements of h have to be identical. The inverted mass ordering is also possible, but as in the classes B and $\tilde{B}$ discussed for matrices with 5 zeros, this is connected with a large spread of parameters. To be precise, the entries b and e of D_2 have to be of order λ^2 and a, d, f of order 1.

The following three classes $\tilde{D}_{1,2,3}$, giving in total 18 matrices, generate a zero in h_{13}:

$$\tilde{D}_1 = \begin{pmatrix} a & 0 & 0 \\ b & d & e \\ 0 & f & 0 \end{pmatrix}, \qquad \tilde{D}_2 \begin{pmatrix} a & 0 & 0 \\ b & d & 0 \\ 0 & e & f \end{pmatrix}, \qquad \tilde{D}_3 = \begin{pmatrix} a & b & 0 \\ d & 0 & e \\ 0 & 0 & f \end{pmatrix}. \tag{3.16}$$

Thus, for them eq. (3.8) holds and the behavior of U_{e3} and $\sin^2 \theta_{23}$ can be seen in figure 1. Similar statements about the neutrino mass scheme as for the classes $D_{1,2,3}$ can be made.

We furthermore find two classes denoted E and $\tilde{E}$, which contain six matrices each. Representants look like:

$$E = \begin{pmatrix} a & b & 0 \\ d & 0 & 0 \\ e & 0 & f \end{pmatrix} \qquad \text{and} \qquad \tilde{E} = \begin{pmatrix} a & 0 & b \\ d & e & 0 \\ f & 0 & 0 \end{pmatrix}. \tag{3.17}$$

Their common feature is a non-vanishing determinant and $J_{CP} = 0$. From the matrices one finds that

$$\frac{h_{12}}{h_{13}} = \frac{h_{22}}{h_{23}} \tag{3.18}$$

for class E while for class $\tilde{E}$ it holds

$$\frac{h_{12}}{h_{13}} = \frac{h_{32}}{h_{33}}. \tag{3.19}$$

Inserting eq. (3.18) in the definition of J_{CP} from eq. (2.6) gives for class E that

$$J_{CP} \propto \mathrm{Im}(h_{12}\, h_{23}\, h_{31}) = \mathrm{Im}(h_{22}\, h_{13}\, h_{31}) = 0\,, \tag{3.20}$$

since h is a hermitian matrix. A similar equation can be written down for class $\tilde{E}$. This explains why there is no CP violation for the classes E and $\tilde{E}$.

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The conditions resulting from eqs. (3.18) and (3.19) can be used to obtain an interesting correlation between the neutrino mixing observables. Starting with class E, the relation eq. (3.18) leads for the normal mass ordering to:

$$
\begin{aligned}
|U_{e3}| &= \frac{\Delta m^2_\odot(\Delta m^2_{\rm A} + m_1^2)\sin 2\theta_{12}\,\tan\theta_{23}}{\Delta m^2_\odot \Delta m^2_{\rm A} + (2\,\Delta m^2_{\rm A} - \Delta m^2_\odot)m_1^2 + \Delta m^2_\odot(\Delta m^2_{\rm A} + m_1^2)\,\cos 2\theta_{12}} \\
&\simeq \begin{cases} \tan\theta_{23}\,\tan\theta_{12} & \text{for } m_1^2 = 0 \\ \frac{1}{2}R\,\sin 2\theta_{12}\,\tan\theta_{23} & \text{for } m_1^2 \gg \Delta m^2_{\odot,A} \end{cases}, \qquad (3.21)
\end{aligned}
$$

where we gave the limits for the normal hierarchical and the quasi-degenerate spectrum. Note that the limit for $m_1^2 \gg \Delta m^2_{\odot,\rm A}$, which corresponds to a quasi-degenerate spectrum, is just the correlation of the parameters eq. (3.8) for a vanishing 13 element of h. The fact that the value of U_{e3} in case of zero m_1 is too large indicates that the matrices of class E lead to a lower limit on the neutrino mass if the spectrum is normally ordered. Indeed, one can solve eq. (3.21) for $m_1^2/\Delta m^2_\odot$, which gives:

$$
\frac{m_1^2}{\Delta m^2_\odot} = \frac{\sin 2\theta_{12}\,\sin\theta_{23} - 2\,\cos^2\theta_{12}\,\cos\theta_{23}\,|U_{e3}|}{2(1 - R\,\sin^2\theta_{12})\,\cos\theta_{23}\,|U_{e3}| - R\,\sin\theta_{23}\,\sin 2\theta_{12}} \qquad (3.22)
$$

$$
\simeq \begin{cases} -\dfrac{1}{R} & \text{for } U_{e3} = 0 \\ \cos\theta_{12}\left(\dfrac{\sin\theta_{12}\,\tan\theta_{23}}{|U_{e3}|} - \cos\theta_{12}\right) & \text{for } |U_{e3}| \gg R \end{cases}. \qquad (3.23)
$$

Consequently, $m_1^2/\Delta m^2_\odot$ is minimized for the largest possible $|U_{e3}|$, and is then given by $m_1^2 \gtrsim \frac{1}{2}\,\Delta m^2_\odot\,\sin 2\theta_{12}\,\tan\theta_{23}/|U_{e3}|$. Putting in numbers gives $m_1 \gtrsim 0.01\,$eV. The mass spectrum can become degenerate when the denominator of eq. (3.22) goes to zero, which occurs when U_{e3} takes the value given in eq. (3.8). This value is about 0.02. Lower values of $|U_{e3}|$ render $m_1^2/\Delta m^2_\odot$ negative and are not possible. Hence, $|U_{e3}| \simeq \frac{1}{2}R\,\sin 2\theta_{12}\,\tan\theta_{23}$ is the lowest possible value for class E and the normal mass ordering.

In case of an inverted mass ordering eq. (3.18) leads to

$$
\begin{aligned}
|U_{e3}| &= \frac{\frac{1}{2}\Delta m^2_\odot m_3^2\,\sin 2\theta_{12}\,\tan\theta_{23}}{\Delta m^2_{\rm A}(m_3^2 + \Delta m^2_{\rm A}) + \Delta m^2_\odot(\Delta m^2_{\rm A} - m_3^2\,\sin^2\theta_{12})} \\
&\simeq \begin{cases} 0 & \text{for } m_3^2 = 0 \\ \frac{1}{2}R\,\sin 2\theta_{12}\,\tan\theta_{23} & \text{for } m_3^2 \gg \Delta m^2_{\odot,A} \end{cases}. \qquad (3.24)
\end{aligned}
$$

In this case there is no lower limit on the smallest mass state because $|U_{e3}|$ goes to zero when m_3 goes to zero. One can again solve eq. (3.24) for the ratio of the smallest neutrino mass m_3 divided by the solar mass squared difference, which gives

$$
\begin{aligned}
\frac{m_3^2}{\Delta m^2_\odot} &= -\frac{2(1-R)\cos\theta_{23}\,|U_{e3}|}{R\left(2(1 + R\,\sin^2\theta_{12})\cos\theta_{23}\,|U_{e3}| - R\,\sin\theta_{23}\,\sin 2\theta_{12}\right)} \\
&\simeq \begin{cases} 0 & \text{for } U_{e3} = 0 \\ -\dfrac{1}{R} & \text{for } U_{e3} \gg R \end{cases}. \qquad (3.25)
\end{aligned}
$$

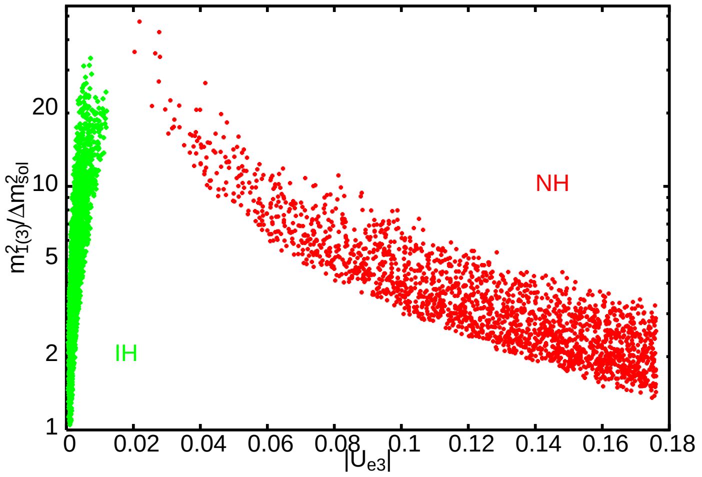

Figure 3: Scatter plot of $|U_{e3}|$ against the ratio of the smallest neutrino mass squared divided by $\Delta m^2_\odot$ for class E, cf. eqs. (3.22), (3.25).

The denominator vanishes when U_{e3} fulfills eq. (3.8), thereby corresponding to quasi-degenerate neutrinos. The expression for $m_3^2/\Delta m^2_\odot$ becomes negative when U_{e3} is larger than roughly 0.02. Hence, $|U_{e3}| \simeq \frac{1}{2} R\, \sin 2\theta_{12}\, \tan\theta_{23}$ is the largest possible value for class E and the inverted mass ordering.

We plot in figure 3 for both mass orderings and class E a scatter plot of $|U_{e3}|$ against the ratio of the smallest neutrino mass squared divided by $\Delta m^2_\odot$. The lower limit of $|U_{e3}|$, roughly 0.02 for the normal mass ordering, occurs for large $m_1^2/\Delta m^2_\odot$, i.e., for quasi-degenerate neutrinos. It is given, as discussed above, by $R/2\, \sin 2\theta_{12}\, \tan\theta_{23}$. The qualitative discussions given above are nicely confirmed by the plot.

For the matrices belonging to class $\tilde{E}$ we have the relation $h_{12}/h_{13} - h_{32}/h_{33} = 0$, which leads to conditions for the observables very similar to eqs. (3.21), (3.24). They are obtained from those equations by replacing $\tan\theta_{23}$ with $\cot\theta_{23}$.

The remaining classes of 4 zero mass matrices predict non-zero U_{e3} and non-vanishing CP violation, which — from the phenomenological point of view — are perhaps the most interesting ones. There are three classes, all of which have a vanishing determinant, i.e., one neutrino is massless:

$$F_1 = \begin{pmatrix} a & 0 & 0 \\ b & d & 0 \\ e & f & 0 \end{pmatrix}, \qquad F_2 = \begin{pmatrix} a & b & 0 \\ d & 0 & 0 \\ e & f & 0 \end{pmatrix}, \qquad F_3 = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ f & 0 & 0 \end{pmatrix}. \tag{3.26}$$

Unfortunately, there is in general no interesting correlation between the parameters and all observables, including the CP phase, can take any value inside their allowed regimes. Again, the reader can easily obtain the rough order of magnitude of the entries in $F_{1,2,3}$ in order to reproduce the approximate textures given in eqs. (2.7), (2.8). However, in contrast to the case of 5 zero entries, the position of the large (i.e., order 1) and small (i.e., order λ or λ^2) terms is not unique and there is some freedom. Examples are given in tables 2

Class	N	Condition	Phenomenology	Order of magnitude NH	Order of magnitude IH
$C = \begin{pmatrix} a & b & d \\ e & 0 & 0 \\ f & 0 & 0 \end{pmatrix}$	3	$\begin{vmatrix} h_{22} & h_{23} \\ h_{32} & h_{33} \end{vmatrix} = 0$	identical to A	—	$\begin{pmatrix} \lambda^2 & 1 & \lambda^2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
$F_1 = \begin{pmatrix} a & 0 & 0 \\ b & d & 0 \\ e & f & 0 \end{pmatrix}$	6	—	$J_{CP} \neq 0$ $U_{e3} \neq 0$	$\begin{pmatrix} \lambda & 0 & 0 \\ \lambda & 1 & 0 \\ \lambda & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ \lambda^2 & 1 & 0 \\ \lambda^2 & 1 & 0 \end{pmatrix}$
$F_2 = \begin{pmatrix} a & b & 0 \\ d & 0 & 0 \\ e & f & 0 \end{pmatrix}$	6	—	same as above	$\begin{pmatrix} \lambda^2 & \lambda & 0 \\ 1 & 0 & 0 \\ 1 & \lambda & 0 \end{pmatrix}$	$\begin{pmatrix} \lambda^2 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & \lambda^2 & 0 \end{pmatrix}$
$F_3 = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ f & 0 & 0 \end{pmatrix}$	6	—	same as above	$\begin{pmatrix} \lambda^2 & \lambda & 0 \\ 1 & \lambda & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \lambda^2 & 1 & 0 \\ 1 & \lambda^2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Table 2: Same as table 1 for matrices with four zero entries and one vanishing mass eigenvalue.

and 3, together with the implied phenomenology and the number of matrices in a given class.

We encounter here the simple example given in the beginning of section 3.1. If the parameters in class F_1 are such that $F_1^\dagger F_1$ is diagonal, we have $V = \mathbb{1}$ and can accommodate the normal hierarchy together with $U_{e3} = 0$, see the discussion after eq. (3.1). This would require in eq. (3.26) that $d\,b^* + f\,e^* = 0$.

We can rule out class C with a non-vanishing value of U_{e3} or when the mass ordering is normal or when laboratory searches or cosmological observations find a signal corresponding to a non-zero neutrino mass. Classes $D_{1,2,3}$ and $\tilde{D}_{1,2,3}$ (which are difficult to distinguish) are ruled out for a value of U_{e3} larger or smaller than $\simeq R/2 \sin 2\theta_{12}$. They can in principle be distinguished from the 5 zero classes B and $\tilde{B}$ because they allow for a non-vanishing smallest neutrino mass. Classes E and $\tilde{E}$ are difficult to distinguish and are excluded when the mass ordering is normal (inverted) and U_{e3} is smaller (larger) than $R/2 \sin 2\theta_{12}$. All the classes C, $D_{1,2,3}$, $\tilde{D}_{1,2,3}$, E and $\tilde{E}$ are ruled out if there is leptonic CP violation. Finally, classes $F_{1,2,3}$ are ruled out if all three neutrinos are found to be massive.

3.4 Three and less texture zeros

As we have seen, already for matrices with 4 zeros the results for the observables can be somewhat arbitrary (see the discussion for the classes $F_{1,2,3}$ after eq. (3.26)). The major part of the successful matrices with 3 or less zeros falls into this category. Counting the number of possibilities for three zero entries, one arrives at 84 of which only 6 generate one

Class	N	Condition	Phenomenology	Order of magnitude NH	Order of magnitude IH
$D_1 = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ d & e & f \end{pmatrix}$	6	$h_{12} = 0$	similar to B $U_{e3} \simeq \frac{R}{2} \sin 2\theta_{12} \cot\theta_{23}$ eq. (3.7), figure 2	$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ \lambda & 1 & \lambda \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda^2 & 1 & \lambda^2 \end{pmatrix}$
$D_2 = \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ e & 0 & f \end{pmatrix}$	6	$h_{12} = 0$	same as above	$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 1 \\ \lambda & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda^2 & 1 \\ \lambda^2 & 0 & 1 \end{pmatrix}$
$D_3 = \begin{pmatrix} a & b & 0 \\ 0 & 0 & d \\ e & 0 & f \end{pmatrix}$	6	$h_{12} = 0$	same as above	$\begin{pmatrix} \lambda & \lambda & 0 \\ 0 & 0 & 1 \\ \lambda & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda & 1 & 0 \\ 0 & 0 & 1 \\ \lambda & 0 & 1 \end{pmatrix}$
$\tilde{D}_1 = \begin{pmatrix} a & 0 & 0 \\ b & d & e \\ 0 & f & 0 \end{pmatrix}$	6	$h_{13} = 0$	similar to $\tilde{B}$ $U_{e3} \simeq \frac{R}{2} \sin 2\theta_{12} \tan\theta_{23}$ eq. (3.8), figure 1	$\begin{pmatrix} \lambda & 0 & 0 \\ \lambda & 1 & \lambda \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ \lambda^2 & 1 & \lambda^2 \\ 0 & 1 & 0 \end{pmatrix}$
$\tilde{D}_2 = \begin{pmatrix} a & 0 & 0 \\ b & d & 0 \\ 0 & e & f \end{pmatrix}$	6	$h_{13} = 0$	same as above	$\begin{pmatrix} \lambda & 0 & 0 \\ \lambda & 1 & 0 \\ 0 & 1 & \lambda \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ \lambda^2 & 1 & 0 \\ 0 & 1 & \lambda^2 \end{pmatrix}$
$\tilde{D}_3 = \begin{pmatrix} a & b & 0 \\ d & 0 & e \\ 0 & 0 & f \end{pmatrix}$	6	$h_{13} = 0$	same as above	$\begin{pmatrix} \lambda & \lambda & 0 \\ \lambda & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \lambda & 0 \\ \lambda^2 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$
$E = \begin{pmatrix} a & b & 0 \\ d & 0 & 0 \\ e & 0 & f \end{pmatrix}$	6	$\frac{h_{12}}{h_{13}} = \frac{h_{22}}{h_{23}}$	NH: $m_1 \gtrsim 0.01$ eV and $U_{e3} \geq \frac{R}{2} \sin 2\theta_{12} \tan\theta_{23}$ eqs. (3.21), (3.22), figure 3 IH: masses free and $U_{e3} \leq \frac{R}{2} \sin 2\theta_{12} \tan\theta_{23}$ eqs. (3.24), (3.25), figure 3	$\begin{pmatrix} \lambda^2 & \lambda & 0 \\ 1 & 0 & 0 \\ 1 & 0 & \lambda^2 \end{pmatrix}$	$\begin{pmatrix} \lambda^2 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & \lambda^2 \end{pmatrix}$
$\tilde{E} = \begin{pmatrix} a & 0 & b \\ d & e & 0 \\ f & 0 & 0 \end{pmatrix}$	6	$\frac{h_{12}}{h_{13}} = \frac{h_{32}}{h_{33}}$	as class E with $\theta_{23} \to \theta_{23} + \pi/4$	$\begin{pmatrix} \lambda^2 & 0 & \lambda \\ 1 & \lambda^2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \lambda^2 & 0 & 1 \\ 1 & \lambda^2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Table 3: Same as table 2 for matrices with four zero entries and no vanishing mass eigenvalue. All classes predict $J_{CP} = 0$.

vanishing eigenvalue. Three of these can accommodate all data and are represented by:

$$\begin{pmatrix} a & b & 0 \\ d & e & 0 \\ f & g & 0 \end{pmatrix} . \tag{3.27}$$

No correlation of observables, except that m_1 or m_3 vanishes, is predicted. This provides a possibility to exclude them. Again, if the parameters in this matrix are such that $m_\nu^\dagger m_\nu$ is diagonal, we have $V = \mathbb{1}$ and can accommodate both hierarchies, compare with the discussion after eq. (3.1).

The remaining 78 matrices without vanishing eigenvalues can be divided into sixteen classes. Two of those, each containing three matrices, fulfill the relation $h_{12} = 0$ and read:

$$\begin{pmatrix} 0 & 0 & a \\ b & d & 0 \\ e & f & g \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & a & b \\ d & 0 & 0 \\ e & f & g \end{pmatrix}.$$

They lead to the same results as the matrices belonging to classes $D_{1,2,3}$ (hence are not distinguishable from the latter), in particular they predict CP conservation, and have the same correlation between U_{e3} and $\sin\theta_{23}$ as given in eq. (3.7) and figure 2. There also exist two classes with $h_{13} = 0$ whose representants are:

$$\begin{pmatrix} 0 & 0 & a \\ b & d & e \\ f & g & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & a & b \\ d & e & f \\ g & 0 & 0 \end{pmatrix}.$$

They lead to the same results as the matrices belonging to classes $\tilde{D}_{1,2,3}$ and consequently are not distinguishable from them and only very hard to distinguish from the above two classes. The presence of CP violation will exclude the last four classes. All the other classes contain six matrices each and are represented by:

$$\begin{aligned}
G_1 &= \begin{pmatrix} 0 & 0 & a \\ 0 & b & d \\ e & f & g \end{pmatrix}, \quad G_2 = \begin{pmatrix} 0 & 0 & a \\ b & d & e \\ 0 & f & g \end{pmatrix}, \quad G_3 = \begin{pmatrix} 0 & a & b \\ 0 & 0 & d \\ e & f & g \end{pmatrix}, \\
G_4 &= \begin{pmatrix} 0 & a & b \\ 0 & d & e \\ f & 0 & g \end{pmatrix}, \quad G_5 = \begin{pmatrix} 0 & a & b \\ d & 0 & e \\ 0 & f & g \end{pmatrix}, \quad G_6 = \begin{pmatrix} 0 & a & b \\ d & 0 & e \\ f & 0 & g \end{pmatrix}, \\
G_7 &= \begin{pmatrix} 0 & a & b \\ d & e & f \\ 0 & 0 & g \end{pmatrix}, \quad G_8 = \begin{pmatrix} a & b & d \\ 0 & 0 & e \\ 0 & f & g \end{pmatrix}, \quad G_9 = \begin{pmatrix} a & b & d \\ 0 & e & f \\ 0 & 0 & g \end{pmatrix}, \\
G_{10} &= \begin{pmatrix} 0 & a & b \\ d & 0 & e \\ f & g & 0 \end{pmatrix}.
\end{aligned} \tag{3.28}$$

None of these reveal simple correlations among the observables.

Similarly, the 36 possible matrices with two texture zeros and the nine ones with only one zero (of course, all of them are allowed) do not show any relations among the mixing angles, the CP-phase and the ratio of the mass squared differences.

3.5 Symmetric matrices with texture zeros

The discussion so far assumed a general structure for the Dirac neutrino mass matrices. In a given model based on some symmetry, however, symmetric matrices might arise. Extending our discussion to the case of symmetric Dirac mass matrices is rather straightforward. It is

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Symmetric Class from eq. (3.28)	Results of [17]	Order of magnitude
$G_1 = \begin{pmatrix} 0 & 0 & a \\ 0 & b & d \\ a & d & e \end{pmatrix}$	Case A_1 only NH $\lvert U_{e3}\rvert \simeq \frac{\sqrt{R}}{2}\cot\theta_{23}\frac{\sin 2\theta_{12}}{\sqrt{\cos 2\theta_{12}}}$	$\begin{pmatrix} 0 & 0 & \lambda \\ 0 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$
$G_2 = \begin{pmatrix} 0 & a & 0 \\ a & b & d \\ 0 & d & e \end{pmatrix}$	Case A_2 only NH $\lvert U_{e3}\rvert \simeq \frac{\sqrt{R}}{2}\tan\theta_{23}\frac{\sin 2\theta_{12}}{\sqrt{\cos 2\theta_{12}}}$	$\begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$
$G_3 = \begin{pmatrix} a & 0 & b \\ 0 & 0 & d \\ b & d & e \end{pmatrix}$	Case B_3 QD; both orderings $\lvert U_{e3}\rvert \simeq R\,\lvert\frac{\cot 2\theta_{23}}{\cos\delta}\rvert\frac{\tan\theta_{12}}{1+\tan^2\theta_{12}}$	$\begin{pmatrix} 1 & 0 & \lambda^2 \\ 0 & 0 & 1 \\ \lambda^2 & 1 & \lambda \end{pmatrix}$
$G_7 = \begin{pmatrix} a & b & 0 \\ b & d & e \\ 0 & e & 0 \end{pmatrix}$	Case B_4 QD; both orderings $\lvert U_{e3}\rvert \simeq R\,\lvert\frac{\tan 2\theta_{23}}{\cos\delta}\rvert\frac{\tan\theta_{12}}{1+\tan^2\theta_{12}}$	$\begin{pmatrix} 1 & \lambda^2 & 0 \\ \lambda^2 & \lambda & 1 \\ 0 & 1 & 0 \end{pmatrix}$
$G_{10} = \begin{pmatrix} a & 0 & b \\ 0 & d & e \\ b & e & 0 \end{pmatrix}$	Case B_2 same as B_4 (or G_7)	$\begin{pmatrix} 1 & 0 & \lambda^2 \\ 0 & \lambda & 1 \\ \lambda^2 & 1 & 0 \end{pmatrix}$
and $\begin{pmatrix} a & b & 0 \\ b & 0 & d \\ 0 & d & e \end{pmatrix}$	Case B_1 same as B_3 (or G_3)	$\begin{pmatrix} 1 & \lambda^2 & 0 \\ \lambda^2 & 0 & 1 \\ 0 & 1 & \lambda \end{pmatrix}$
$\begin{pmatrix} a & b & d \\ b & 0 & e \\ d & e & 0 \end{pmatrix}$	Case C QD; both orderings $\lvert U_{e3}\rvert \simeq \frac{\cot 2\theta_{12}\,\cot 2\theta_{23}}{\cos\delta}$	$\begin{pmatrix} 1 & \lambda^2 & \lambda \\ \lambda^2 & 0 & 1 \\ \lambda & 1 & 0 \end{pmatrix}$

Table 4: Symmetric Dirac mass matrices from eq. (3.28) together with the corresponding two zero Majorana mass matrices from refs. [17]. Class C from [17] would correspond to a symmetric Dirac mass matrix with two zero entries.

known from the discussion of ref. [17] that Majorana mass matrices, which are necessarily symmetric, can only have two independent zero entries. It turns out now that if the three zero Dirac mass matrices in the classes G_i (see eq. (3.28)) are symmetric, they can reproduce six of the seven allowed two zero Majorana mass matrices from refs. [17]. In the language of those references, the two zero Majorana mass matrix A_1 corresponds to the symmetric form of matrix G_1 from eq. (3.28). Analogously, it is easy to see that G_2, G_3, G_7 from eq. (3.28) correspond to A_2, B_3, B_4 from refs. [17], respectively. The matrices belonging to class G_{10} reproduce cases B_1 and B_2. We display this correspondence in table 4. Case C from [17] would correspond to a symmetric mass matrix with two zero entries, whose discussion we omitted in this work, since in general no interesting correlation of variables arises from this possibility. The phenomenological consequences of two zero Majorana mass matrices are rather well known [17] and we have nothing to add here and refer to those works for details of their phenomenological implications. Nevertheless, we included the approximate formulae out of which the phenomenology follows in the table.

3.6 Unsuccessful matrices

We finish this section with a few comments on matrices which are unsuccessful in explaining the neutrino data.

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Class	N	Condition	Phenomenology
matrices with one vanishing eigenvalue			
$\begin{pmatrix} 0 & b & 0 \\ 0 & d & 0 \\ a & e & 0 \end{pmatrix}$	6	$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 0$	NH: $U_{\tau 1} = 0$ IH: $U_{\tau 3} = 0$
$\begin{pmatrix} a & b & 0 \\ d & 0 & 0 \\ 0 & e & 0 \end{pmatrix}$	6	$h_{23} = 0$	$\lvert U_{e3}\rvert$ too large
$\begin{pmatrix} a & 0 & 0 \\ b & d & e \\ f & 0 & 0 \end{pmatrix}$	3	$\begin{vmatrix} h_{11} & h_{13} \\ h_{31} & h_{33} \end{vmatrix} = 0$	NH: $U_{\mu 1} = 0$ IH: $U_{\mu 3} = 0$
matrices with all $m_i \neq 0$			
$\begin{pmatrix} a & b & d \\ 0 & 0 & e \\ 0 & f & 0 \end{pmatrix}$	6	$h_{23} = 0$	similar to second matrix in this table
$\begin{pmatrix} a & 0 & 0 \\ b & 0 & d \\ e & f & 0 \end{pmatrix}$	6	$\frac{h_{11}}{h_{12}} = \frac{h_{31}}{h_{32}}$	$\lvert U_{e3}\rvert$ too large

Table 5: Typical matrices with four and five zeros unable to accommodate the data. For discussion see section 3.6.

In case of 5 zeros in m_ν, there are three categories of matrices which do not work. First, there are 90 matrices which lead — independently of the choice of the four complex parameters — to the vanishing of two mixing angles in U. In the second category, there are twelve matrices which generate a zero (12)– or (13)–subdeterminant in h. A vanishing of the (12)–subdeterminant leads for the normal hierarchy ($m_1 = 0$) to $U_{\tau 1} = 0$ and for the inverted hierarchy ($m_3 = 0$) to $U_{\tau 3} = 0$. Analogously, if the (13)–subdeterminant vanishes, one ends up with $U_{\mu 1} = 0$ for the normal mass ordering and $U_{\mu 3} = 0$ for the inverted ordering. All these conditions are of course not compatible with the data. In the last category, there are six matrices predicting $h_{23} = 0$. Phenomenologically $h_{23} = 0$ leads to a too large value for U_{e3} for the normal as well as the inverted hierarchy. Hence, 108 out of 126 possible matrices with 5 zeros are unsuccessful in explaining the neutrino data.

For matrices with four zeros we find that about half of the 126 possible matrices, i.e., 57 matrices, do not work. 27 matrices — 9 of which have a non-vanishing determinant — are not successful since two of the mixing angles are zero. Furthermore, three matrices generate a vanishing (12)–subdeterminant in h and further three ones a vanishing (13)–subdeterminant. Apart from these there are 18 matrices leading to $h_{23} = 0$. Finally, there are six matrices fulfilling the equation $h_{11}/h_{12} = h_{31}/h_{32}$. In this case $|U_{e3}|$ turns out to be always bounded from below by $\simeq 0.3$ and hence one cannot accommodate the data.

In table 5 we give typical examples of matrices with 4 and 5 zero entries which are not able to explain the data.

Only 9 of the possible 84 three zero textures do not work. Three of them enforce two mixing angles to be zero and the six other ones generate $h_{23} = 0$.

4. Realization of the textures

In this section we indicate models which might give rise to the textures found to be able to reproduce the data. They originate either from the Froggatt-Nielsen mechanism [28] — explaining the hierarchy among the non-zero matrix elements — or from more complicated flavor symmetries. As an example, we choose the global non-abelian discrete symmetry $D_4 \times Z_2$ which is broken at the electroweak scale by enlarging the Higgs sector through scalars transforming non-trivially under the horizontal symmetry.[2] A global continuous symmetry broken at high energies is assumed to be the origin of this discrete symmetry. Rather than working out the details of the models, we merely reproduce the structure of the mass matrices, so that this section should be understood as a proof of principle for the Dirac neutrino mass textures under consideration. One still faces the problem of explaining the relative smallness of neutrino masses with respect to the charged lepton masses. To achieve this, we have to rely on some tuning of the parameters of the models based on the discrete symmetry. In the model based on the Froggatt-Nielsen mechanism, we can explain the relative smallness by appropriate choice of charges and by working with two scalar fields, whose vacuum expectation values display a hierarchy.

4.1 Mass textures from the non-abelian discrete symmetry $D_4 \times Z_2$

The group D_4 has already been discussed as a possible flavor symmetry by several authors [30–33]. Other dihedral and two-valued dihedral groups have also been considered in the literature [34]. Here we discuss D_4 for the first time in the context of Dirac neutrinos. Mathematical details of D_4 are delegated to the appendix A.

4.1.1 Obtaining class A from $D_4 \times Z_2$

Consider the Dirac neutrino mass matrix to be of the form of eq. (3.4) and the charged lepton mass matrix to be diagonal. As shown in the following, this setup can be realized in the framework of the Standard Model extended by the non-abelian flavor symmetry $D_4 \times Z_2$. The left-handed leptons, right-handed charged leptons and neutrinos are assigned to be:

$$\left\{ L_e, \begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix} \right\} \sim \left\{ \underline{\mathbf{1}}_1^+, \underline{\mathbf{2}}^+ \right\},$$
$$\left\{ e_R, \begin{pmatrix} \mu_R \\ \tau_R \end{pmatrix} \right\} \sim \left\{ \underline{\mathbf{1}}_1^-, \underline{\mathbf{2}}^- \right\},$$

[2] Of course there are several ways to realize texture zeros in mass matrices, see, for instance [29].

$$\{(\nu_e)_R, (\nu_\mu)_R, (\nu_\tau)_R\} \sim \left\{\underline{\mathbf{1}}_{\mathbf{1}}^-, \underline{\mathbf{1}}_{\mathbf{2}}^+, \underline{\mathbf{1}}_{\mathbf{2}}^-\right\}.$$

The left-handed lepton doublets are $L_e = \binom{\nu_e}{e}_L$ and analogously for L_μ and L_τ. The Higgs sector of the model contains 5 SM-like $\mathrm{SU}(2)_L$ doublets, two of which form a two-dimensional representation under D_4 and the three other ones a one-dimensional one:

$$\phi_1^- \sim \underline{\mathbf{1}}_{\mathbf{1}}^-, \qquad \phi_2^+ \sim \underline{\mathbf{1}}_{\mathbf{2}}^+, \qquad \phi_4^- \sim \underline{\mathbf{1}}_{\mathbf{4}}^-, \qquad \begin{pmatrix} \psi_1^+ \\ \psi_2^+ \end{pmatrix} \sim \underline{\mathbf{2}}^+,$$

where $\underline{\mathbf{i}}_{\mathbf{j}}$ denotes the j^{th} representation of dimension i of D_4 and the superscript $^\pm$ the transformation property under Z_2. Therefore, a field ϕ^+ transforms under Z_2 as ϕ^+, whereas ϕ^- transforms as $-\phi^-$.

Using real representation matrices for the two-dimensional representation, this leads to the following tree level mass matrices

$$m_\nu = \begin{pmatrix} \kappa_1 \langle \phi_1^- \rangle & \kappa_2 \langle \phi_2^+ \rangle & 0 \\ 0 & \kappa_3 \langle \psi_2^+ \rangle & 0 \\ 0 & \kappa_3 \langle \psi_1^+ \rangle & 0 \end{pmatrix}$$

and

$$m_\ell = \begin{pmatrix} \kappa_4 \langle \phi_1^- \rangle & 0 & 0 \\ 0 & \kappa_5 \langle \phi_1^- \rangle + \kappa_6 \langle \phi_4^- \rangle & 0 \\ 0 & 0 & \kappa_5 \langle \phi_1^- \rangle - \kappa_6 \langle \phi_4^- \rangle \end{pmatrix},$$

which corresponds to the Dirac mass matrix of class A with a diagonal charged lepton mass matrix. The Yukawa couplings are denoted with κ_i and the Higgs fields $\phi_i^\pm$ and $\psi_i^\pm$ have acquired their vacuum expectation values (vevs). Obtaining a hierarchy for the entries of the mass matrices requires for this and the next two examples, in general a detailed analysis of the Higgs potential, which is beyond the scope of the present work. Still one can give some order-of-magnitude estimates for the vevs and Yukawa couplings. This illustrates the tuning required to explain the relative smallness of neutrino and charged lepton masses. Invoking the framework in a scenario involving higher dimensions might explain the overall suppression of neutrino masses along the lines of [5, 6]. This is well beyond the scope of our work, which is supposed to deal mainly with the phenomenological consequences of texture zeros. Let us give an example of the required values of couplings and vevs: with v being the electroweak scale one can choose $\langle \phi_2^+ \rangle = \mathcal{O}(10^{-14})\, v$, $\langle \psi_1^+ \rangle = \mathcal{O}(10^{-12})\, v$,$\langle \psi_2^+ \rangle = \mathcal{O}(10^{-12})\, v$, $\langle \phi_1^- \rangle = \mathcal{O}(10^{-2})\, v$, $\langle \phi_4^- \rangle = \mathcal{O}(10^{-2})\, v$ for the vevs and $\kappa_1 = \mathcal{O}(10^{-10})$, $\kappa_4 = \mathcal{O}(10^{-4})$, κ_2, κ_3, κ_5, $\kappa_6 = \mathcal{O}(1)$. Note that with these choices there has to exist one further Higgs doublet whose vev is of the order of v. For example, one might introduce a Higgs field transforming as $\underline{\mathbf{1}}_{\mathbf{1}}^+$, which cannot couple to the leptons because of their assignments under $D_4 \times Z_2$, but can give adequate masses to the quarks.

4.1.2 Obtaining class B from $D_4 \times Z_2$

To obtain class B from the group $D_4 \times Z_2$ we choose the following assignments for the leptons:

$$\left\{ \begin{pmatrix} L_e \\ L_\mu \end{pmatrix}, L_\tau \right\} \sim \left\{ \underline{\mathbf{2}}^+, \underline{\mathbf{1}}_{\mathbf{1}}^+ \right\},$$
$$\left\{ \begin{pmatrix} e_R \\ \mu_R \end{pmatrix}, \tau_R \right\} \sim \left\{ \underline{\mathbf{2}}^-, \underline{\mathbf{1}}_{\mathbf{1}}^- \right\},$$
$$\left\{ \begin{pmatrix} (\nu_e)_R \\ (\nu_\mu)_R \end{pmatrix}, (\nu_\tau)_R \right\} \sim \left\{ \underline{\mathbf{2}}^+, \underline{\mathbf{1}}_{\mathbf{2}}^- \right\},$$

while for the Higgs fields we now require 6 doublets:

$$\phi_1^- \sim \underline{\mathbf{1}}_{\mathbf{1}}^-, \qquad \phi_2^+ \sim \underline{\mathbf{1}}_{\mathbf{2}}^+, \qquad \phi_3^+ \sim \underline{\mathbf{1}}_{\mathbf{3}}^+, \qquad \phi_4^- \sim \underline{\mathbf{1}}_{\mathbf{4}}^-, \qquad \begin{pmatrix} \psi_1^+ \\ \psi_2^+ \end{pmatrix} \sim \underline{\mathbf{2}}^+.$$

These choices lead to the following mass matrices:

$$m_\nu = \begin{pmatrix} 0 & \kappa_2 \langle \phi_2^+ \rangle - \kappa_3 \langle \phi_3^+ \rangle & 0 \\ \kappa_2 \langle \phi_2^+ \rangle + \kappa_3 \langle \phi_3^+ \rangle & 0 & 0 \\ \kappa_1 \langle \psi_1^+ \rangle & \kappa_1 \langle \psi_2^+ \rangle & 0 \end{pmatrix}$$

and

$$m_\ell = \begin{pmatrix} \kappa_4 \langle \phi_1^- \rangle + \kappa_5 \langle \phi_4^- \rangle & 0 & 0 \\ 0 & \kappa_4 \langle \phi_1^- \rangle - \kappa_5 \langle \phi_4^- \rangle & 0 \\ 0 & 0 & \kappa_6 \langle \phi_1^- \rangle \end{pmatrix}.$$

The hierarchy among the elements in the mass matrices can be reached in a similar way as explained in the previous example. Note that here we have different vevs appearing in the matrices governing m_ν and m_ℓ, which simplifies the issue.

4.1.3 Obtaining class D_1 from $D_4 \times Z_2$

A texture belonging to class D_1 can be reproduced by the following assignments for the leptons which is very similar to the assignment used to create textures belonging to the class B:

$$\left\{ \begin{pmatrix} L_e \\ L_\mu \end{pmatrix}, L_\tau \right\} \sim \left\{ \underline{\mathbf{2}}^+, \underline{\mathbf{1}}_{\mathbf{1}}^+ \right\},$$
$$\left\{ \begin{pmatrix} e_R \\ \mu_R \end{pmatrix}, \tau_R \right\} \sim \left\{ \underline{\mathbf{2}}^-, \underline{\mathbf{1}}_{\mathbf{1}}^- \right\},$$
$$\left\{ \begin{pmatrix} (\nu_e)_R \\ (\nu_\mu)_R \end{pmatrix}, (\nu_\tau)_R \right\} \sim \left\{ \underline{\mathbf{2}}^+, \underline{\mathbf{1}}_{\mathbf{1}}^- \right\}.$$

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For the Higgs particles we choose:

$$\phi_1^+ \sim \underline{\mathbf{1}}_{\mathbf{1}}^+ , \qquad \phi_1^- \sim \underline{\mathbf{1}}_{\mathbf{1}}^- , \qquad \phi_4^+ \sim \underline{\mathbf{1}}_{\mathbf{4}}^+ , \qquad \phi_4^- \sim \underline{\mathbf{1}}_{\mathbf{4}}^- , \qquad \begin{pmatrix} \psi_1^+ \\ \psi_2^+ \end{pmatrix} \sim \underline{\mathbf{2}}^+ ,$$

yielding the mass matrices

$$m_\nu = \begin{pmatrix} \kappa_1 \langle \phi_1^+ \rangle + \kappa_2 \langle \phi_4^+ \rangle & 0 & 0 \\ 0 & \kappa_1 \langle \phi_1^+ \rangle - \kappa_2 \langle \phi_4^+ \rangle & 0 \\ \kappa_3 \langle \psi_1^+ \rangle & \kappa_3 \langle \psi_2^+ \rangle & \kappa_4 \langle \phi_1^- \rangle \end{pmatrix}$$

and

$$m_\ell = \begin{pmatrix} \kappa_5 \langle \phi_1^- \rangle + \kappa_6 \langle \phi_4^- \rangle & 0 & 0 \\ 0 & \kappa_5 \langle \phi_1^- \rangle - \kappa_6 \langle \phi_4^- \rangle & 0 \\ 0 & 0 & \kappa_7 \langle \phi_1^- \rangle \end{pmatrix} .$$

Again we can reproduce the hierarchy among the elements and the relative smallness of the neutrino masses with an interplay of vevs and Yukawa couplings.

4.2 Mass textures from the Froggatt-Nielsen mechanism

In this subsection we show how one can generate the textures under consideration in a supersymmetric framework by assuming the existence of two global horizontal flavor symmetries denoted $\mathrm{U}(1)_A$ and $\mathrm{U}(1)_B$. As an example, we generate class A of section 3.2. In addition to the two global U(1) symmetries, we require the presence of the cyclic symmetry Z_3 acting only on the charged leptons to preserve their mass matrix to be diagonal. Under this Z_3 symmetry, the right– and left-handed charged leptons transform according to $\sim \left\{1, \omega, \omega^2\right\}$, where $\omega = e^{2\pi i/3}$. The $\mathrm{U}(1)_{A,B}$ are both broken at high-energy scales M_A and M_B, respectively. There are two scalar fields σ and ρ, which are gauge group singlets. The field σ (ρ) carries charge -1 under $\mathrm{U}(1)_A$ ($\mathrm{U}(1)_B$) and, by acquiring a vev, whose magnitude is of the order M_A (M_B), breaks the horizontal symmetry spontaneously. Small parameters associated to this framework are given by the ratio of those vevs with some high scale M, which corresponds to the mass of the additional heavy vector-like fermions, which are integrated out. The U(1)-charges are chosen as follows:

$$\begin{aligned} \{L_e, L_\mu, L_\tau\} &\sim \{(3_A, 0_B), (5_A, 0_B), (5_A, 0_B)\} , \\ \{e_R, \mu_R, \tau_R\} &\sim \{(8_A, 0_B), (7_A, 0_B), (5_A, 0_B)\} , \\ \{(\nu_e)_R, (\nu_\mu)_R, (\nu_\tau)_R\} &\sim \{(3_A, 6_B), (5_A, 6_B), (0_A, 6_B)\} . \end{aligned}$$

The MSSM Higgs doublets H_u and H_d are assumed to have zero charges under both U(1) symmetries. For the fields ρ and σ we choose the following possibilities for their charge assignment and their vevs:

$$\sigma \sim (-1_A, 0_B) \qquad \text{with } \frac{\langle \sigma \rangle}{M} \equiv \lambda \sim 0.22 , \tag{4.1}$$

$$\rho \sim (0_A, -1_B) \qquad \text{with } \frac{\langle \rho \rangle}{M} \equiv \epsilon \sim 10^{-2} . \tag{4.2}$$

The relative smallness of $\langle\rho\rangle/M$ is required to explain the hierarchy between the charged lepton and neutrino masses, whereas $\langle\sigma\rangle/M$ is responsible for the mass hierarchy between the generations of the charged leptons and neutrinos, respectively. We end up with:

$$m_\nu = \mathcal{O}(\epsilon^6) \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda^2) & 0 \\ 0 & \mathcal{O}(1) & 0 \\ 0 & \mathcal{O}(1) & 0 \end{pmatrix} \langle H_u\rangle \qquad \text{and} \qquad m_\ell = \mathcal{O}(1) \begin{pmatrix} \mathcal{O}(\lambda^5) & 0 & 0 \\ 0 & \mathcal{O}(\lambda^2) & 0 \\ 0 & 0 & \mathcal{O}(1) \end{pmatrix} \langle H_d\rangle \,.$$

Note that the zeros in m_ν are preserved from being filled, since we are working in a SUSY framework, in which the conjugated field of σ, which would carry the charges $(1_A, 0_B)$, is absent ("SUSY zeros").

5. Conclusions, summary and final remarks

Apart from theoretical prejudices, we have no incontrovertible evidence for the Majorana nature of neutrinos. If they are Dirac particles, just like all other known fermions, one might be interested in the form of the Dirac mass matrix m_ν implied by current data. In particular, the minimal allowed structure of m_ν might be of special interest. Here, the situation is analyzed in terms of putting as many zeros in m_ν as possible. As a proof of principle, we have also outlined possibilities to obtain the successful Dirac neutrino mass matrix textures from models based on either discrete non-abelian symmetries or on a Froggatt-Nielsen scenario. We found that up to 5 zero entries in m_ν are allowed by current data. All of those candidates imply CP conservation, one massless neutrino and their phenomenology is summarized in table 1. Most successful matrices with 4 zero entries also predict CP conservation, their physical implications are summarized in tables 2 and 3. Apart from a few exceptions, which then reproduce the phenomenology of already discussed matrices with 5 or 4 zeros, all matrices having 3 or less zeros imply no correlation between the neutrino mixing observables. Typically the inverted hierarchy requires that there is a spread between the non-zero entries in m_ν of two orders of magnitude, whereas the normal hierarchy can be successfully obtained with a rather mild hierarchy. We would like to remark that some matrices predict $\theta_{13} = 0$, but none of them implies in general $\theta_{23} = \pi/4$. This would require typically that two of the non-zero entries in m_ν are identical. It is finally interesting to note that there are basically only three significantly different predictions coming from Dirac mass matrices with zero entries. Those are the inverted hierarchy with zero U_{e3} and the two possibilities given in eqs. (3.7), (3.8) and (3.21), (3.24). Note that once there is an interesting correlation between observables, CP is automatically conserved. The characteristic value of U_{e3} which can play the role of a discriminator between those possibilities, is given by $|U_{e3}| \simeq \frac{1}{2} R \sin 2\theta_{12}$.

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A. Group theory of D_4

The group theory is given by the following character table, generator relations and matrices [35]:

	classes				
	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
G	$\mathbb{1}$	A^2	B	AB	A
$^\circ\mathcal{C}_i$	1	1	2	2	2
$^\circ\mathrm{h}_{\mathcal{C}_i}$	1	2	2	2	4
$\underline{\mathbf{1}}_{\mathbf{1}}$	1	1	1	1	1
$\underline{\mathbf{1}}_{\mathbf{2}}$	1	1	-1	1	-1
$\underline{\mathbf{1}}_{\mathbf{3}}$	1	1	-1	-1	1
$\underline{\mathbf{1}}_{\mathbf{4}}$	1	1	1	-1	-1
$\underline{\mathbf{2}}$	2	-2	0	0	0

Here we denoted with $\underline{\mathbf{1}}_{\mathbf{1,2,3,4}}$ and $\underline{\mathbf{2}}$ the irreducible representations of D_4, G are the generators of the group, $^\circ\mathcal{C}_i$ is the order of the class $\mathcal{C}_i$ and $^\circ\mathrm{h}_{\mathcal{C}_i}$ is the order of the elements in class $\mathcal{C}_i$, where $i = 1, \ldots, 5$. The generators A and B for $\underline{\mathbf{2}}$ are

$$\mathrm{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad \mathrm{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and fulfill the generator relations

$$\mathrm{A}^4 = \mathbb{1}\,, \qquad \mathrm{B}^2 = \mathbb{1} \qquad \text{and} \qquad \mathrm{ABA} = \mathrm{B}\,.$$

From the character table one can read off the Kronecker products:

$\times$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$
$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$
$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$
$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$
$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$

$$\underline{\mathbf{2}} \times \underline{\mathbf{1}}_{\mathbf{i}} = \underline{\mathbf{2}} \quad \forall\, \mathrm{i}\,, \qquad [\underline{\mathbf{2}} \times \underline{\mathbf{2}}] = \underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{4}}\,, \qquad \{\underline{\mathbf{2}} \times \underline{\mathbf{2}}\} = \underline{\mathbf{1}}_{\mathbf{3}}\,,$$

where $[\underline{\mathbf{n}} \times \underline{\mathbf{n}}]$ denotes the symmetric and $\{\underline{\mathbf{n}} \times \underline{\mathbf{n}}\}$ the anti-symmetric part of the product $\underline{\mathbf{n}} \times \underline{\mathbf{n}}$.

The Clebsch-Gordan coefficients for the products $\underline{\mathbf{2}} \times \underline{\mathbf{1}}_{\mathbf{i}}$ are then with $\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} \sim \underline{\mathbf{2}}$ and $t \sim \underline{\mathbf{1}}_{\mathbf{i}}$:

$$\begin{matrix} i=1 & i=2 & i=3 & i=4 & \\ \begin{pmatrix} s_1 t \\ s_2 t \end{pmatrix} & \begin{pmatrix} s_2 t \\ s_1 t \end{pmatrix} & \begin{pmatrix} s_2 t \\ -s_1 t \end{pmatrix} & \begin{pmatrix} s_1 t \\ -s_2 t \end{pmatrix} & \sim \underline{\mathbf{2}} \end{matrix}$$

and for the product $\underline{\mathbf{2}} \times \underline{\mathbf{2}}$ for $\begin{pmatrix} s_1 \\ s_2 \end{pmatrix}, \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \sim \underline{\mathbf{2}}$:

$$\frac{(s_1 t_1 + s_2 t_2)}{\sqrt{2}} \sim \underline{\mathbf{1}}_{\mathbf{1}} \qquad \text{and} \qquad \frac{(s_1 t_2 + s_2 t_1)}{\sqrt{2}} \sim \underline{\mathbf{1}}_{\mathbf{2}}\,,$$
$$\frac{(s_1 t_2 - s_2 t_1)}{\sqrt{2}} \sim \underline{\mathbf{1}}_{\mathbf{3}} \qquad \text{and} \qquad \frac{(s_1 t_1 - s_2 t_2)}{\sqrt{2}} \sim \underline{\mathbf{1}}_{\mathbf{4}}\,.$$

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